

# Practical Applications of Artificial Intelligence for Robotics

## Day 2: Kinematics & Manipulation

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School of Computing Technologies  
RMIT University



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ESSAI July 2023





## Acknowledgement of Country

RMIT University acknowledges the people of the Woi wurrung and Boon wurrung language groups of the eastern Kulin Nation on whose unceded lands we conduct the business of the University.

RMIT University respectfully acknowledges their Ancestors and Elders, past and present.

RMIT also acknowledges the Traditional Custodians and their Ancestors of the lands and waters across Australia where we conduct our business.

Artwork 'Luwaytini' by Mark Cleaver, Palawa



# Motivation

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# Where is the ball?

*Worked Example*



# Frames of Reference

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# Frames of Reference & Poses

To describe poses a Frame is defined by:

1. Origin Point
2. 3D axes orientation
3. Following Right-hand Rule

A Pose is the tuple:

$[position, orientation]$

$[x, y, z, roll, pitch, yaw]$

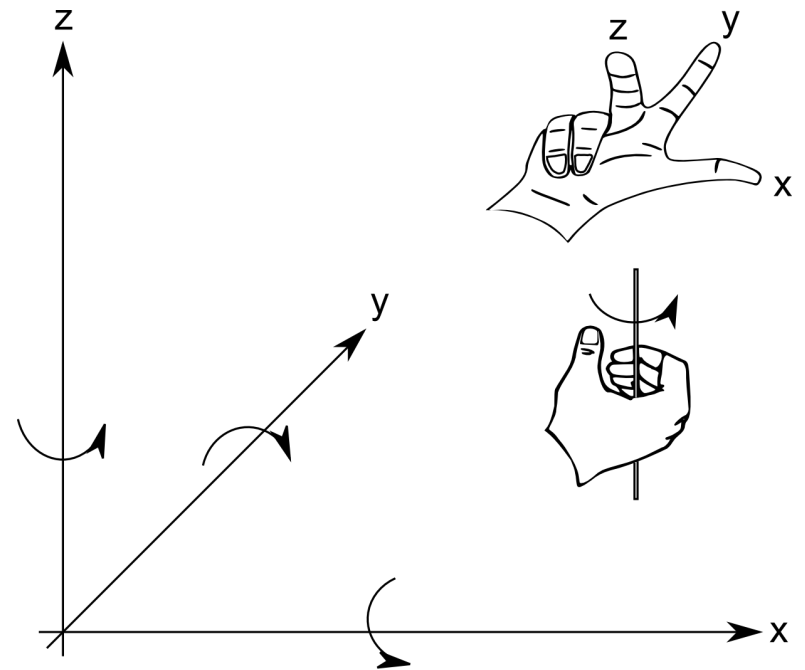


Image: Correll, 2022, introduction  
to Autonomous Robots

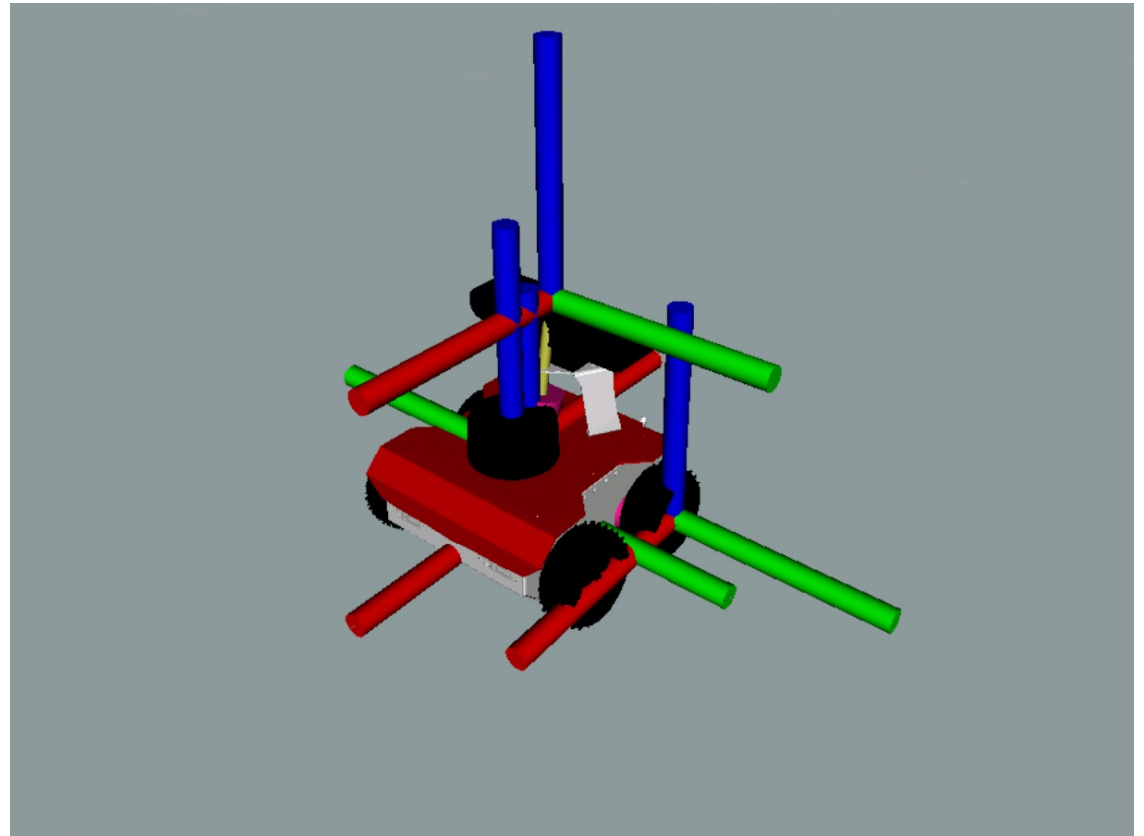




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# ROSBot Frames & Conventions

X – Red  
Y – Green  
Z - Blue



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# ROSBot Frames & Conventions

Conventionally:

- The “global” frame is /map
  - The x/y-axis parallel to the ground-place
  - The z-axis is vertical
- The default frame of robot is /base\_link with
  - The x-axis is the ‘forward’ direction of the robot
  - The z-axis is vertical
- The default frame of a device is /<device>\_base\_link





# Transforming Between Frames (links)

Defined by Linear Algebra Transformation with Homography Matrices

Point:

$${}^A P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Translation:

$$T = \begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotations

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





## — Transforming Between Frames (links)

Transforming a Point, Vector (or Pose) between frames:

$${}^W P = {}^W_R T \times {}^R P$$

Transformation Matrices can be multiplied, but order matters

$${}^W_R T = T_x \times T_y \times R_x$$





## — Transforming Between Frames (links)







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## Defining Links

Links (between frames) may be:

- Static – such as fixed displacements of robot geometry
- Actuator – that take dynamic values from actuator (joint) measurements of the robot. Typically these are rotational links.
- Mapping/Odometry – that describe the transform from the “world” frame to a moving robot/object position. Typically these are a combination of a translation and rotation



# Transform Tree

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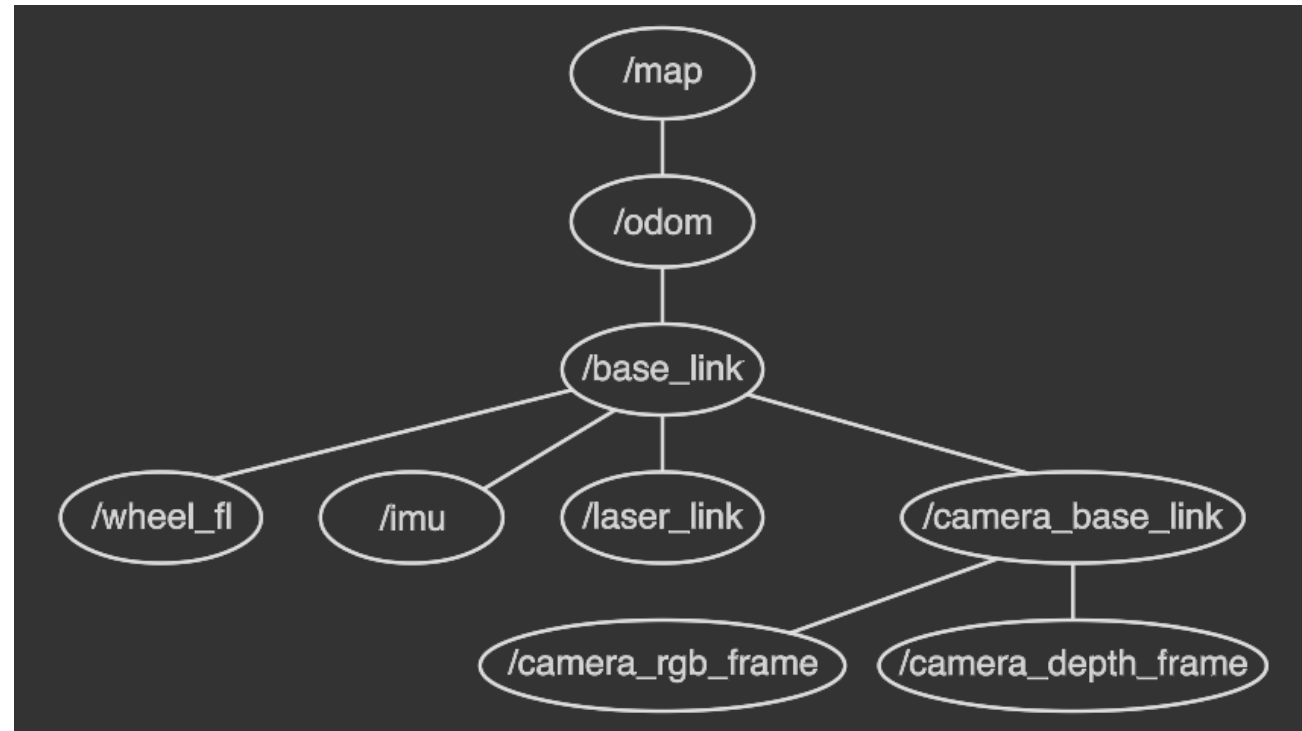


## — Defining a Complete Transform Tree





# — Defining a Complete Transform Tree



# Forward Kinematics

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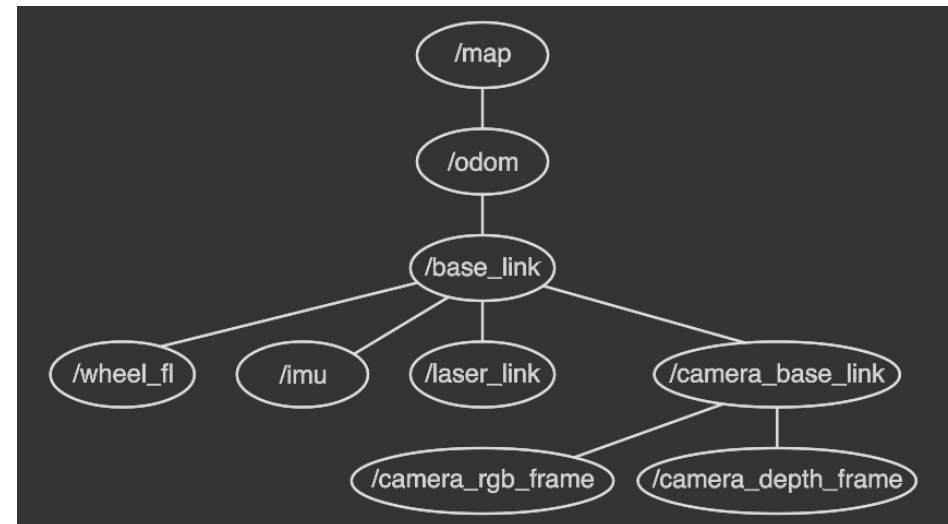
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# Definition of Forward Kinematics

Forward kinematics is the “forward” application of the TF tree where:

- All TFs are known, that is, all joint angles are known
- Transform a point, vector, pose in one frame into another frame

$$\begin{aligned} \text{camera\_rgb\_frame } P &= \text{base\_link } T^{-1} \times \\ &\text{laser\_link } T \times \\ &\text{base\_link } T \times \\ &\text{camera\_base\_link } T \times \\ &\text{camera\_base\_link } T \times \\ &\text{camera\_rgb\_frame } T \times \\ &\text{laser\_link } P \end{aligned}$$



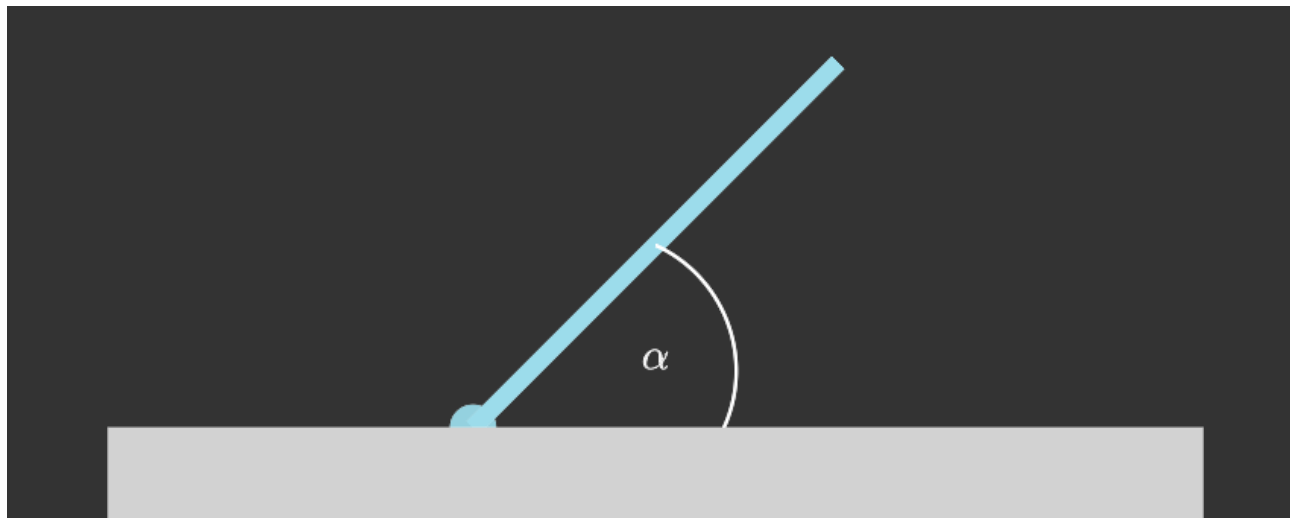


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# Single Joint Example

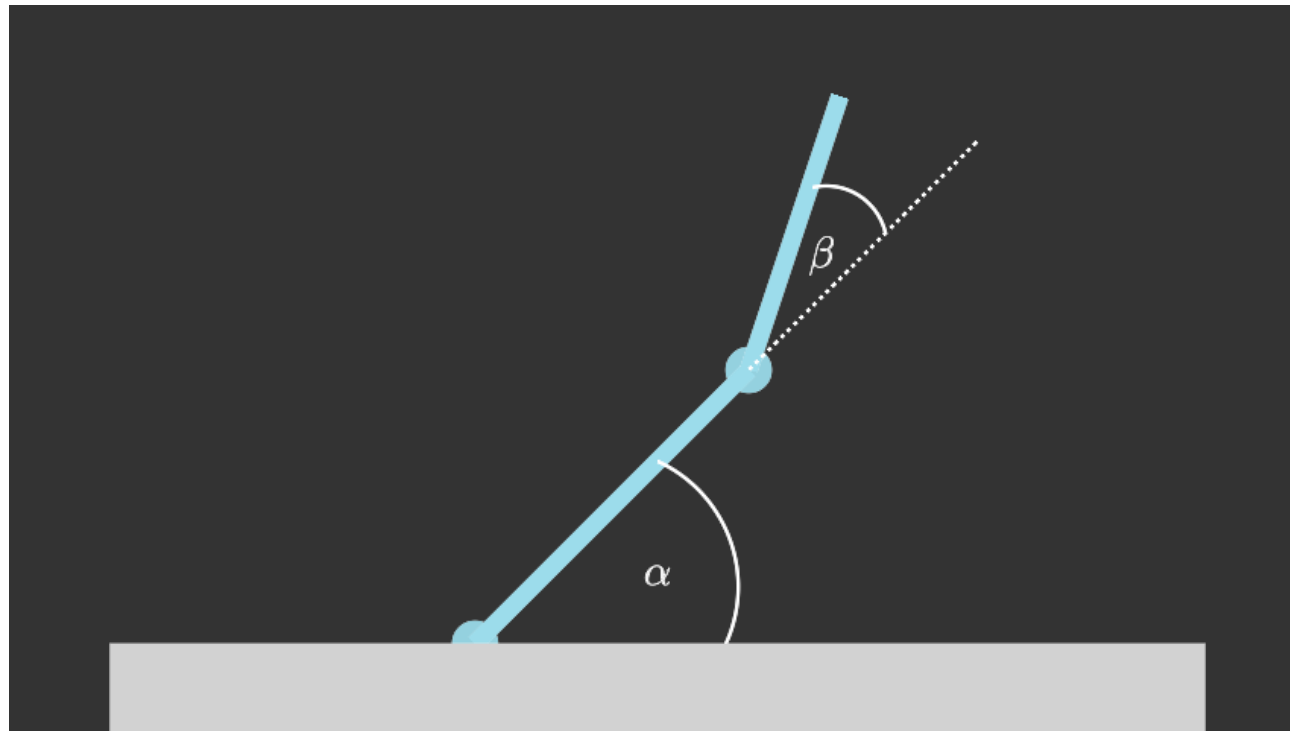
For a forward kinematic setup, what is the “world” position of the arm end-effector:

- The arm base is fixed at the “World” origin
- The angle  $\alpha$  is known



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# Double Joint Example



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# Transforms using the ROSBot in ROS

*Worked Example*

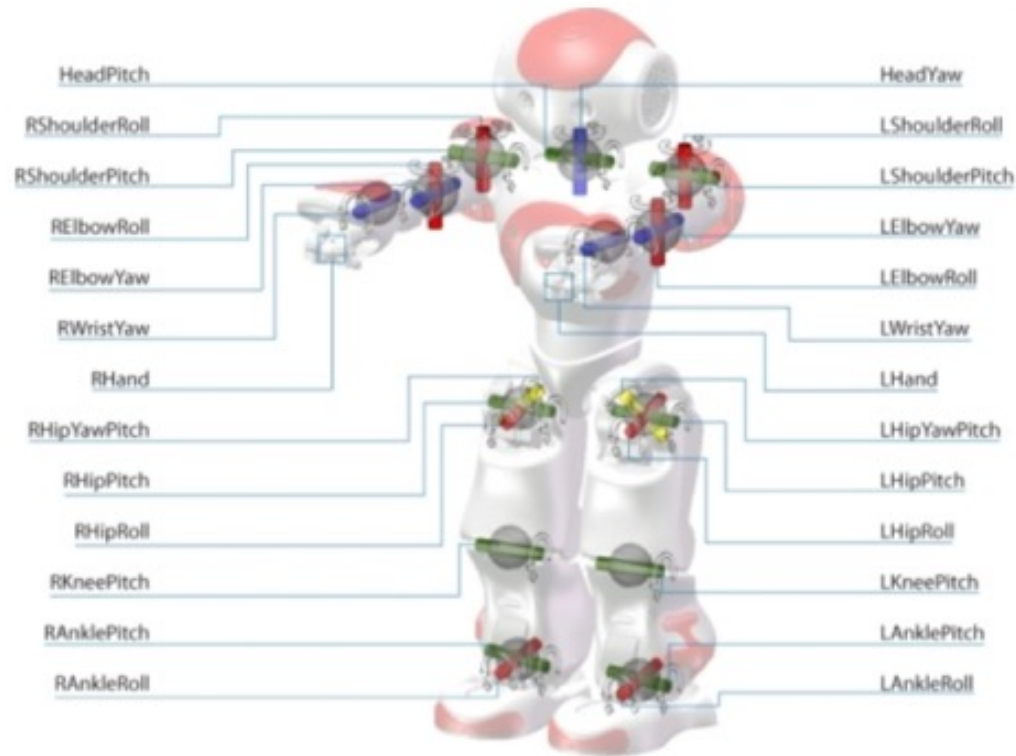




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# Nao Kinematics

A more complex robot has more transformation frames. But, this doesn't impact the overall mathematics on kinematic transforms





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## Real-time Kinematic Problems

Refer back to the worked example on Transforms using ROSbot in ROS.

The general issues to be aware of for any kinematics:

- Asynchronous updates of dynamic links
- Only approximate correspondence to sensor inputs, which also update asynchronously at a different rate to actuators
- Delays between TF update, processing, behaviour generation and final joint actuation





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# Kinematic Prediction

It is possible to use Machine Learning to predict the “current” kinematic state of a robot using “past” TF observations. ML structures include:

- Simple Feed-Forward neural network with a sliding window
- Recurrent Neural Network

For example, Jan Fiedler and Tim Laue (2023, University of Bremen) *“Neural Network-based Joint Angle Prediction for the NAO Robot”*

The prediction does improve stability of walk and repones to falling

Limitations of prediction:

- Highly dependent on the actuation task (ie walk/kick)
- Introduce further error for bad prediction





# Inverse Kinematics

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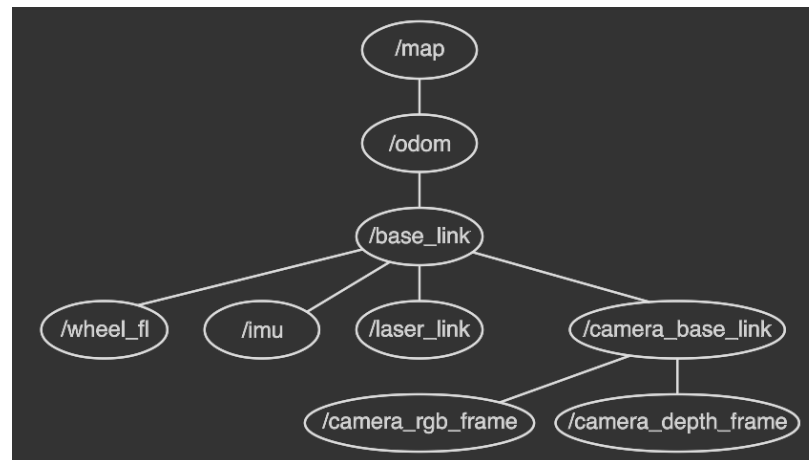
# Definition of Inverse Kinematics

Inverse kinematics is the computation of required joint states to achieve a end-effector position:

- Static TFs are known
- Dynamic TFs must be computed

Requires solving for the transform of the forward kinematic equation

$${}^A P = {}_B^A T \times {}^B P$$

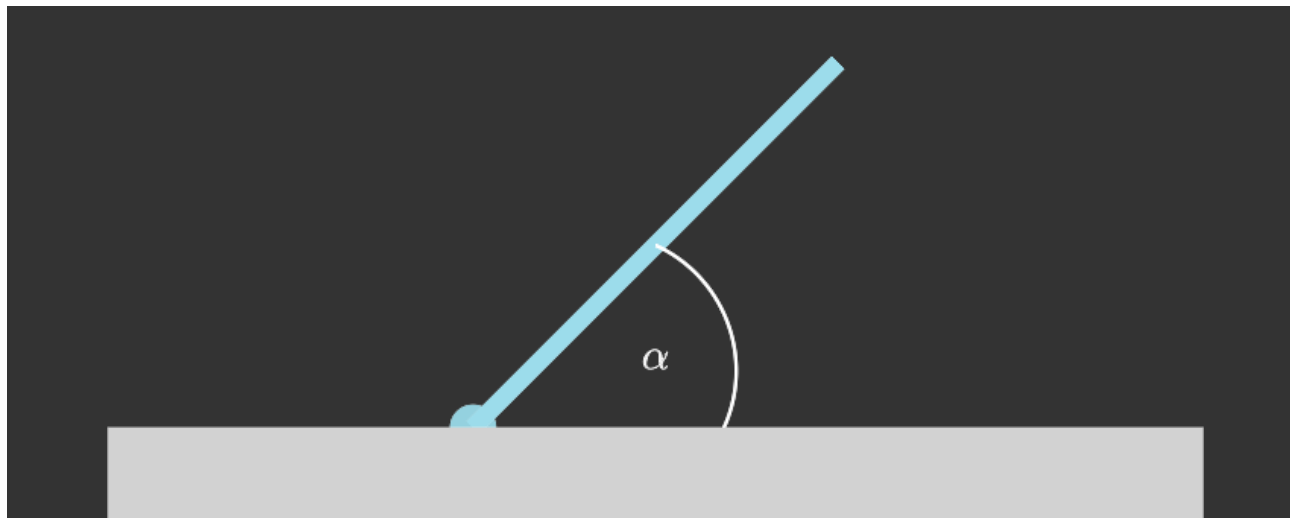


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# Single Joint Example

For an inverse kinematic setup, what is required angle,  $\alpha$ , for:

- The arm end-effector to be at:  $P = [3, 2]$
- Where, the arm base is fixed at the “World” origin



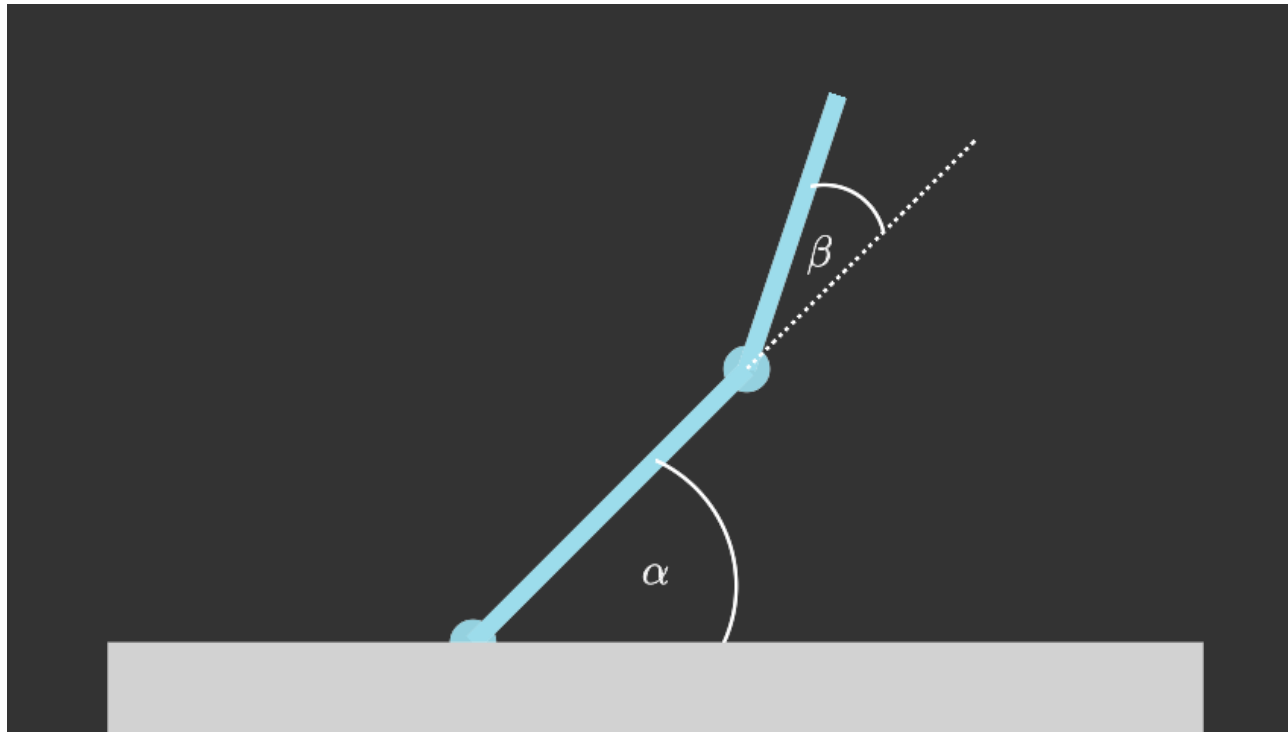


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# Double Joint Example

What is required angles,  $\alpha$  and  $\beta$ , for:

- The arm end-effector to be at:  $P = [3, 2]$





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## Closed Form Solvability

It is possible to solve the equation using closed form methods:

- Observe that robotics systems are highly non-linear
- Analytical solutions can be found, depending on the complexity of the non-linear system
- Likely to have multiple solutions
- A robotic system should have at least 6DOF

*Worked example in Matlab with the Puma560 arm*



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# Closed Form Solvability

Limitations:

- Computation for complex kinematic chains
- Unsolvable solutions for under-actuated systems
- Multiple solutions for over-actuated system
- Singularities, if using Euclidean Transforms





# Alternative Transform Representation

The transforms present are Euclidean Affine Transforms.  
Alternative transform representations include:

- DH-Parameters, common in Mechanical Engineering
- Quaternions (for rotations)

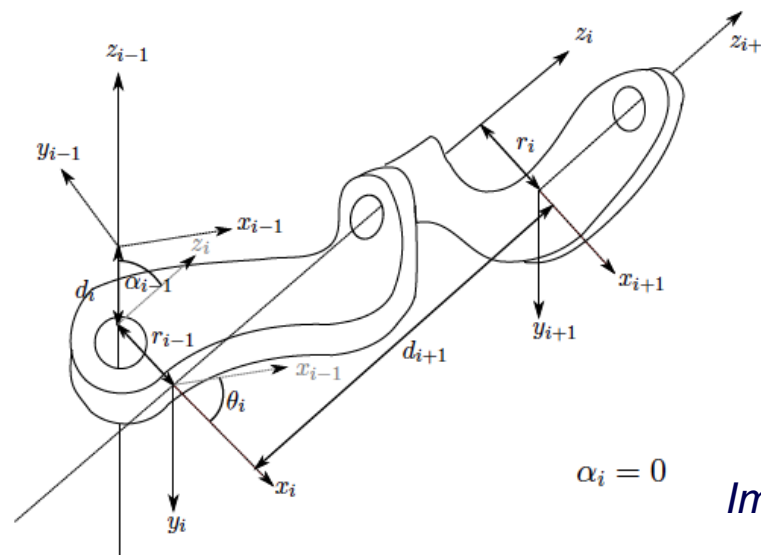


Image: Correll, 2022, introduction  
to Autonomous Robots





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## Approximate Solving via the Jacobian

Approximate closed-form solutions can be more efficient and “reasonable”:

- The kinematics define a multi-dimensional configuration space
- Finding the closed-form solution is the same as finding the minimum of this configuration space.
- This can be done by employing a technique similar to gradient descent to find the minimum
- That is, finding the derivative of the configuration space





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# Approximate Solving via the Jacobian

The Jacobian matrix,  $J$ , is all partial derivatives of a systems kinematics:

$$J = \frac{\partial f}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \dots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \dots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$

For an analytical solution, solving still requiring inverting the Jacobian, which may not be possible. However:

- An approximation to inverting  $J$  is:  $J^+ = \frac{J^T}{J \cdot J^T} = \frac{1}{J}$
- This can be used for an iterative convergence method:  $\Delta j = J^+ \epsilon$
- Where  $\epsilon$  is the error between the desired and actual position





# Motion Control

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## Types of control

The general form of control for a robotic actuation system uses:

- Position control
- Velocity control
- Acceleration control

Rigid-body motion mathematically formalises a robot's motion:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \boldsymbol{\tau}_g(\mathbf{q}) = \boldsymbol{\tau}$$





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# Rigid-Body Motion

Rigid-body motion mathematically formalises a robot's motion:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \boldsymbol{\tau}_g(\mathbf{q}) = \boldsymbol{\tau}$$

But what is this mean?

- Control of is a function of time, that is, we are controlling a dynamic motion, not an instantaneous snapshot
- Control requires effective measurement of the joint parameters
- This motion is a function of all joint parameters
- The desired control can be computed by solving the motion equation
- There are many different motion equations depending on the robot





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# Open-Loop Motion Control

Rigid-body motion mathematically formalises a robot's motion:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \boldsymbol{\tau}_g(\mathbf{q}) = \boldsymbol{\tau}$$

This is what we have already looked at with inverse kinematics.



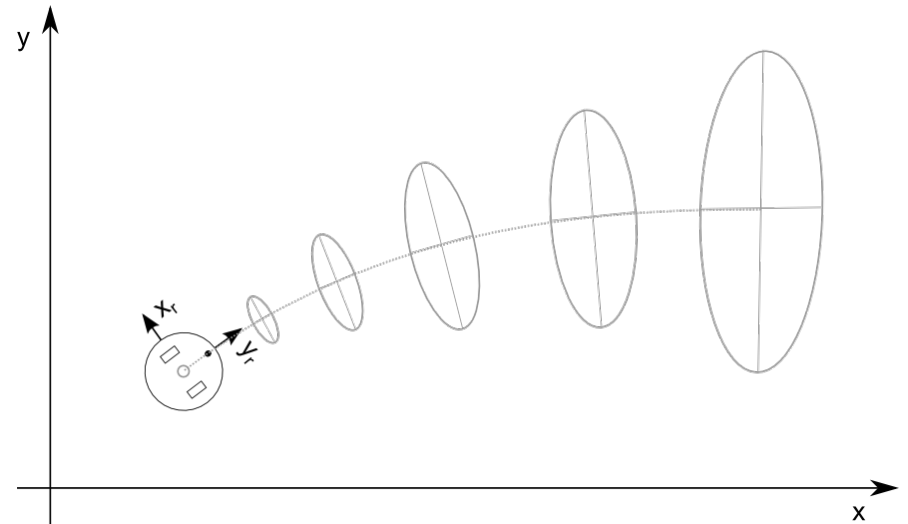


# Practical Issues of Motion Control

Aside from the issue of solveability, open-loop control has issues with:

- Sensor noise
- Instantaneous actuation error
- Accumulative error
- Drift

*Worked Example with ROSBot*



*Image: Correll, 2022, introduction to Autonomous Robots*

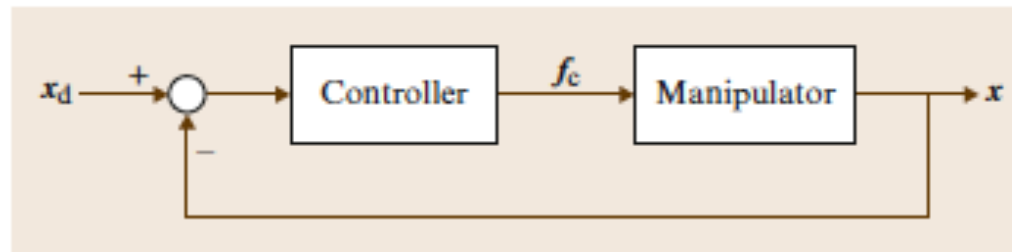


# Closed-Loop Control

Closed-loop control actively monitors the error between a desired set-point of the actuator and actual-position. The motion-control (position, velocity and/or acceleration) is then adjusted based on the error.

This means the actuator is “servo-ed” into position gradually.

Closed-loop control can also be used in-place of inverse kinematic solving.



*Image: Siciliano & Khatib (Eds), 2016,  
Springer Handbook of Robotics*



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# PID Control

The most common form of closed-loop control is PID-control.

The general PID controller equation is:

$$\tau = K_p e_q + K_i \int e_q dt + K_d \frac{e_q}{dt}$$

Where:

- P – Proportion of the instantaneous error
- I – Integral of the cumulative error
- D – Derivative of the instantaneous change in the error
- Each 'K' term is the 'Gain' of how much each PID term should be weighted



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# PID Control: Online example

The general PID controller equation is:

$$\tau = K_p e_q + K_i \int e_q dt + K_d \frac{e_q}{dt}$$

Online PID example: <http://grauonline.de/alexwww/ardumower/pid/pid.html>





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# PID Control: ROSBot

The general PID controller equation is:

$$\tau = K_p \mathbf{e}_q + K_i \int \mathbf{e}_q dt + K_d \frac{d\mathbf{e}_q}{dt}$$

*ROSBot worked example of PID Control*



# Motion Planning

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# Motion Planning

Simple closed-loop control assumed that the robot actuators are capable of moving through all intermediate ranges without conflict.

For a complex actuation system, such as a robot arm, this assumption is not true, due to:

- Self collisions
- Environment collisions





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# Motion Planning: Example







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# Motion Planning

Motion planning overcomes this problem, by finding a sequence of intermediate positions for actuators before the end-position is reached.

A motion plan is the sequence of joint movements to move the joints from one position to another position.

Motion planning is one of the hardest problems now in robotics due to:

- Need for precise fine-grained motor control motion
- The complex nature of solving motion equations





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# Example with the Puma560

*Worked example in Matlab with the Puma 560*

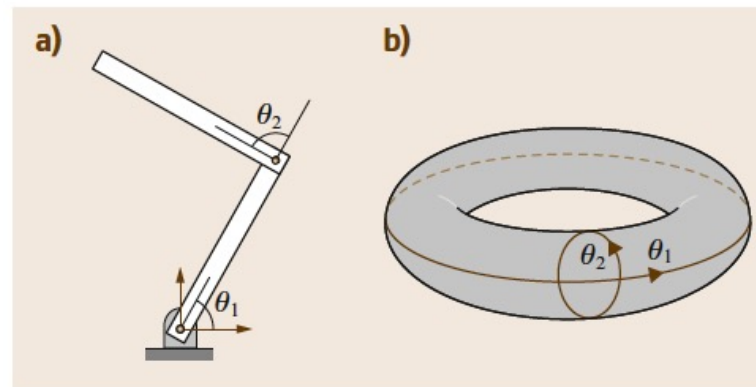


# Configuration Spaces

Motion planning can be solved analytically, but this is computationally expensive.

An different approach to motion planning uses Configuration Spaces.

A configuration space described all valid actuator positions. When visualised, this is the actuator space, *not* the 3D space of the environment.



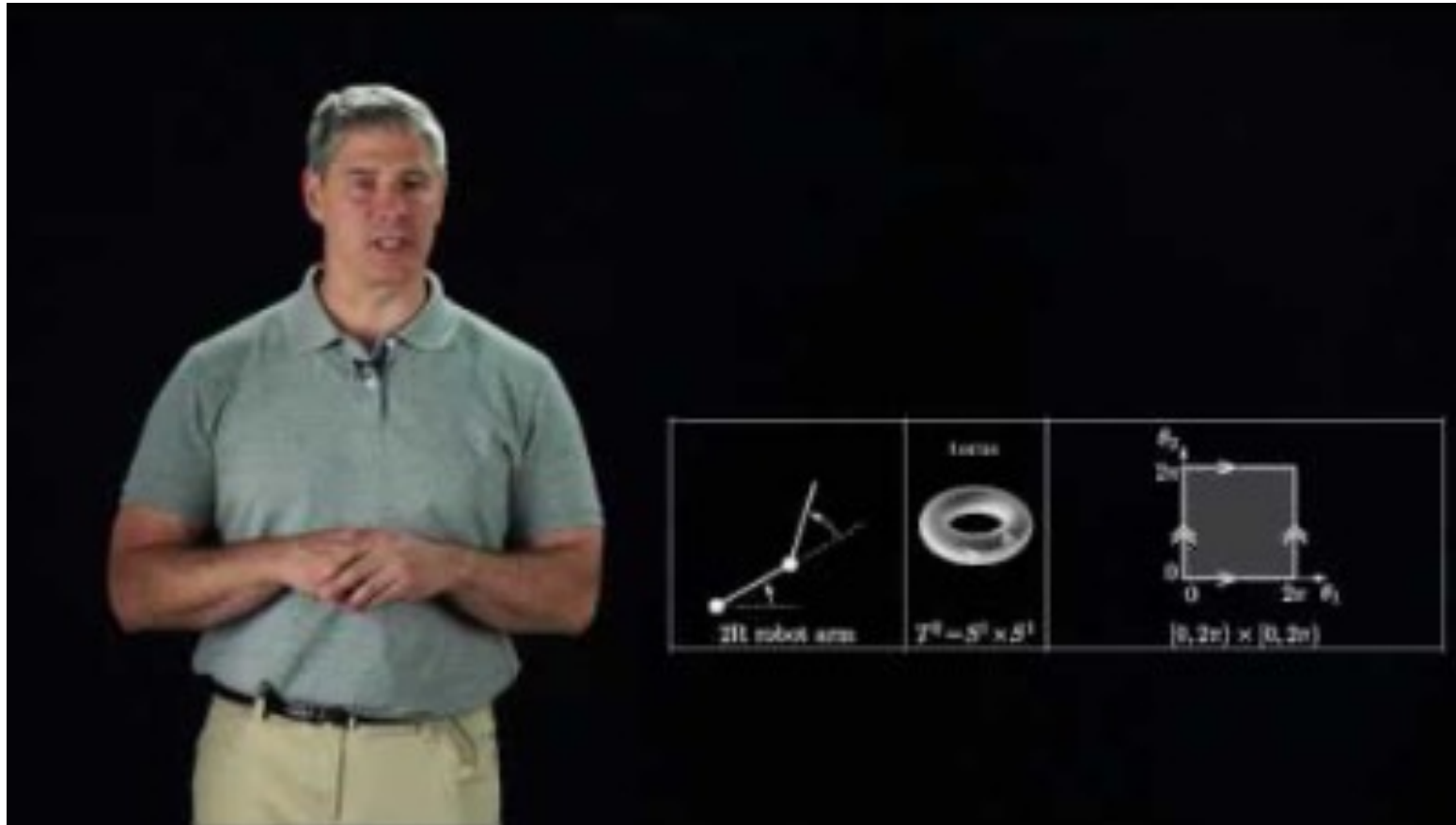
**Fig. 7.2** (a) A two-joint planar arm in which the links are pinned and there are no joint limits. (b) The C-space

*Image: Siciliano & Khatib (Eds), 2016,  
Springer Handbook of Robotics*



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# Configuration Spaces



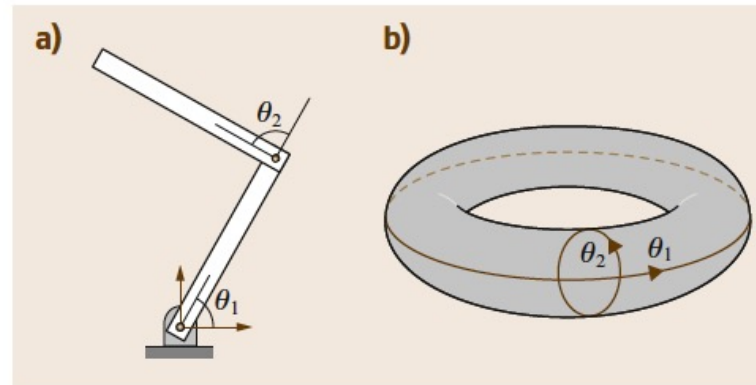


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# Configuration Spaces

Configuration spaces:

- May have complex shapes for self collisions
- Include boundaries (or breaks) for environment collisions



**Fig. 7.2** (a) A two-joint planar arm in which the links are pinned and there are no joint limits. (b) The C-space

*Image: Siciliano & Khatib (Eds), 2016,  
Springer Handbook of Robotics*



# Sampling Based Motion Planning

A common approach to Motion Planning is to randomly sample the configuration space and build a traversable graph through the space. Algorithm:

1. Initialise a Graph,  $G$ , with the starting & end configuration.
2. Randomly sample a configuration,  $\alpha(i)$ , from within the configuration space, and add as a vertex to  $G$
3. Find all vertices in  $G$  within a neighbourhood of  $\alpha(i)$  and connect the vertices if possible
4. Repeat (2) & (3)
5. Terminate when a path is found and at least  $N$  vertices are added to  $G$

Movelt, the ROS package, uses sampling based approaches, and you can visualise the planning

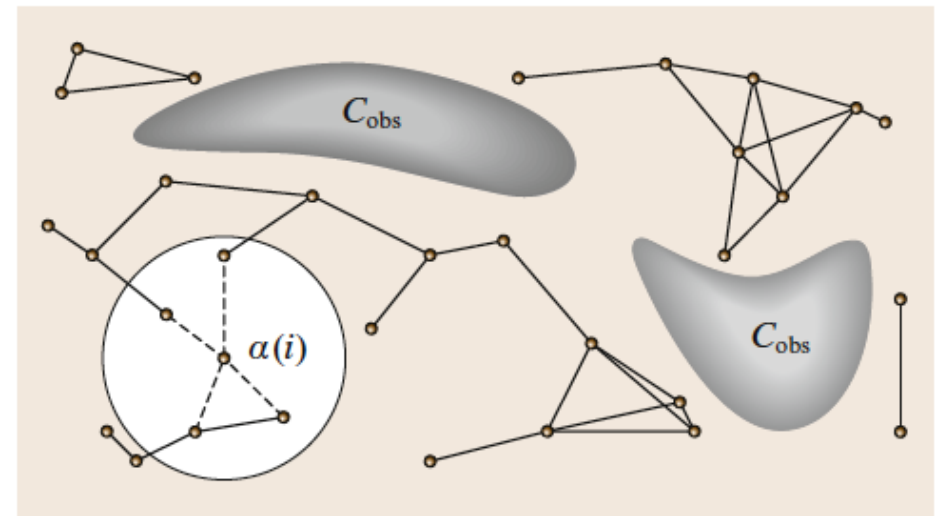


Image: Siciliano & Khatib (Eds), 2016, Springer Handbook of Robotics



# Learning Motion

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# Learning Motion

Machine Learning in both simulation and online-learning is leveraged to learn complex motion. This could be a course to itself.

The typical set-up uses various forms of reinforcement learning. The RL problems is defined as:

$$RL := \langle S, A, T, R, \gamma \rangle$$

Where:

- S – State Space (discretised, continuous)
- A – Action Set (discrete, continuous)
- T – Transition Function
- R – Reward Function
- $\gamma$  – Discount Factor







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# Learning Motion

The typical set-up uses various forms of reinforcement learning. The RL problems is defined as:

$$RL := \langle S, A, T, R, \gamma \rangle$$

Again, entire courses can be devoted to variations on this definition, along with structures for the RL agent. Wide varieties of RL methods have been investigated in both simulation and online learning:

- Value Iteration
  - Q-Learning
  - Deep Q-Learning
  - Actor-Critic
- ... to name a few





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# Learning Motion: Issues

Reinforcement Learning methods encounter the same issues:

- Iterations required for convergence
- Balancing Exploration vs Exploitation
- Appropriate definition of the RL problem

But, additional challenges are faced:

- Transferring simulated behaviours to real-systems
- Non-deterministic actions (not just probabilistic)
- Noise
- Actual time required for learning online
- Manual labour configuration of online trials
- Hardware failures



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## Learning Motion: Examples



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# Learning Motion: Examples (DeepMind)



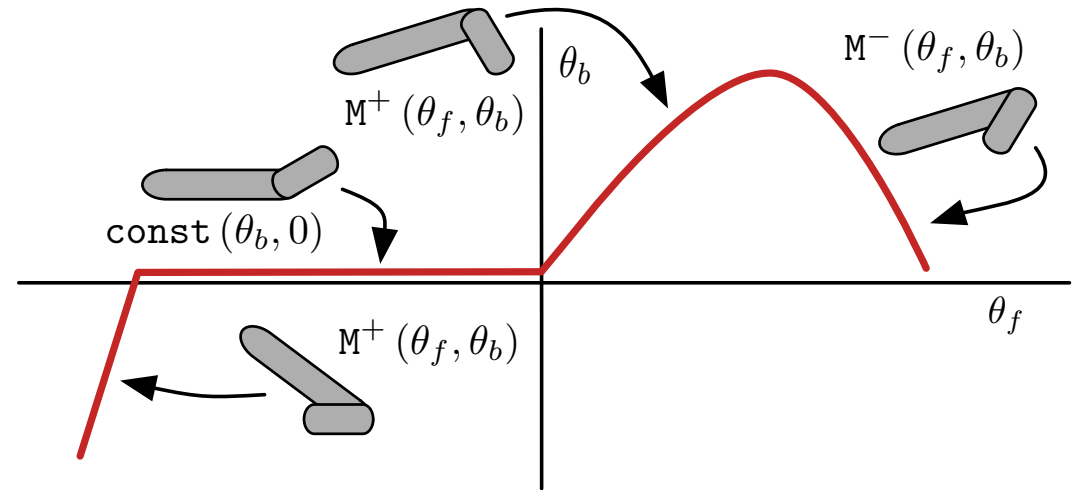
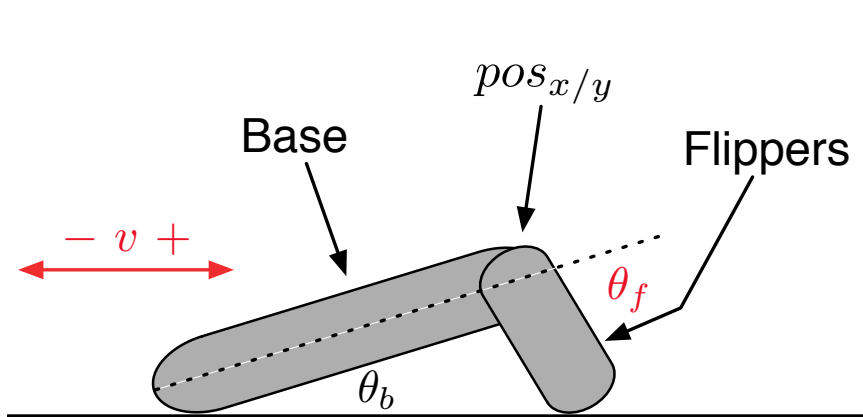


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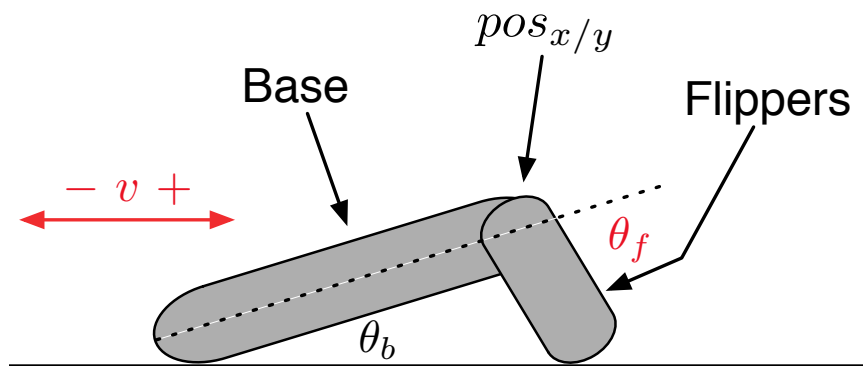
# Combining Planning and Reinforcement Learning for Learning Locomotion



# Negotiator Robot Model



# Naive Reinforcement Learning



$$RL := \langle S, A, T, R, \gamma \rangle$$

$$S := [\theta_b, \theta_f, v, pos_x, pos_y, \dots]_{discrete}$$

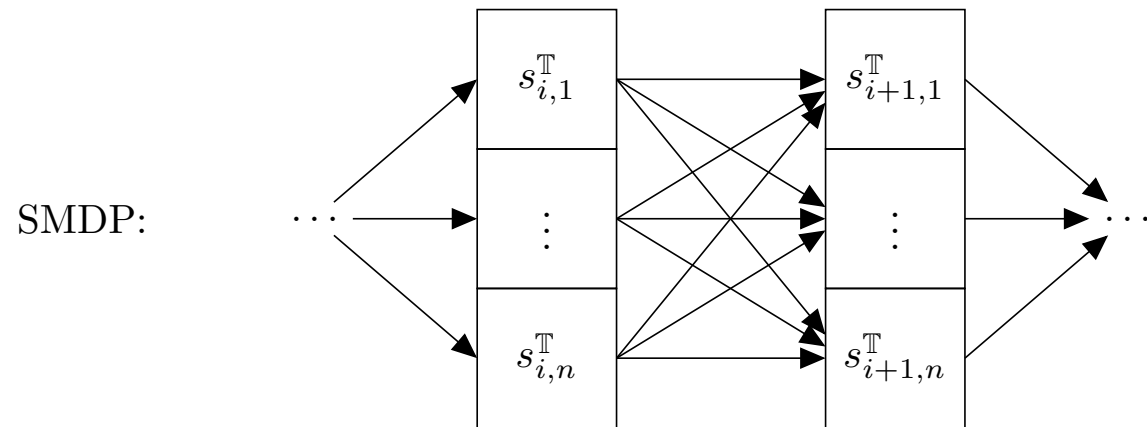
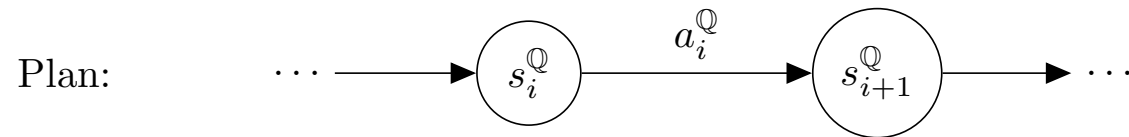
$$A := [noop, inc_{\theta_f}, dec_{\theta_f}, inc_v, dec_v]$$

$$R := \begin{cases} 100 & \text{if at goal,} \\ 0 & \text{if not at goal} \end{cases}$$



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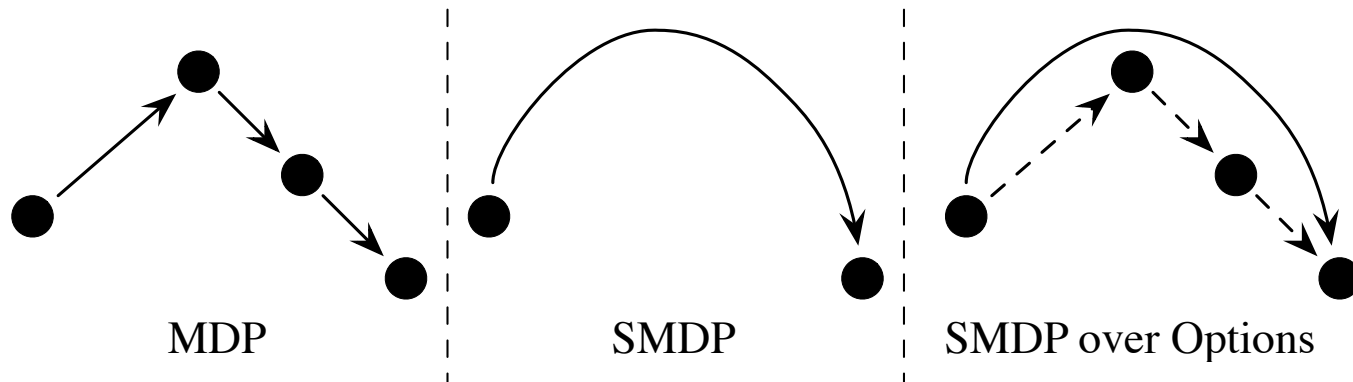
# Planning into Reinforcement Learning



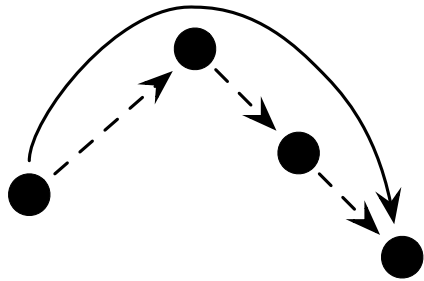


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# SMDP Over Options

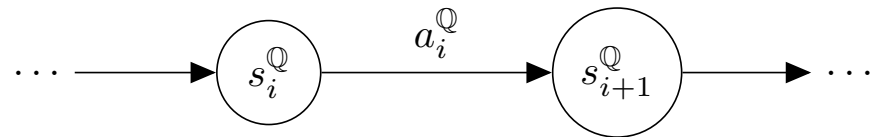


# Planning into Reinforcement Learning

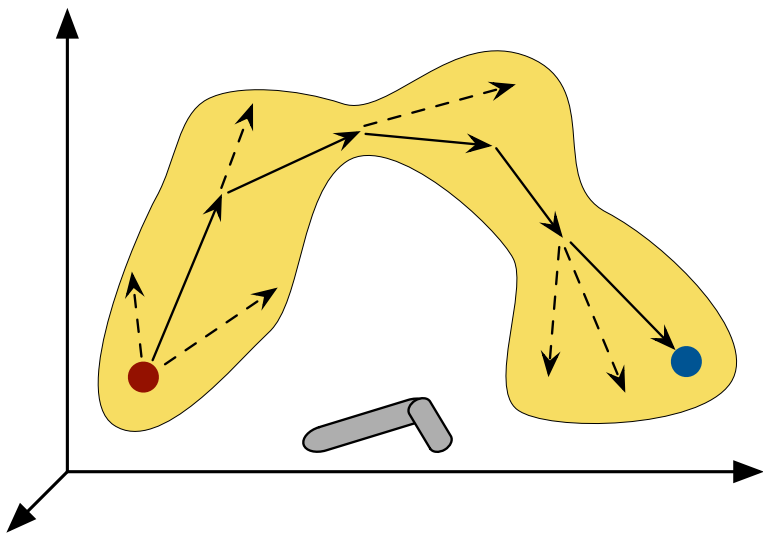
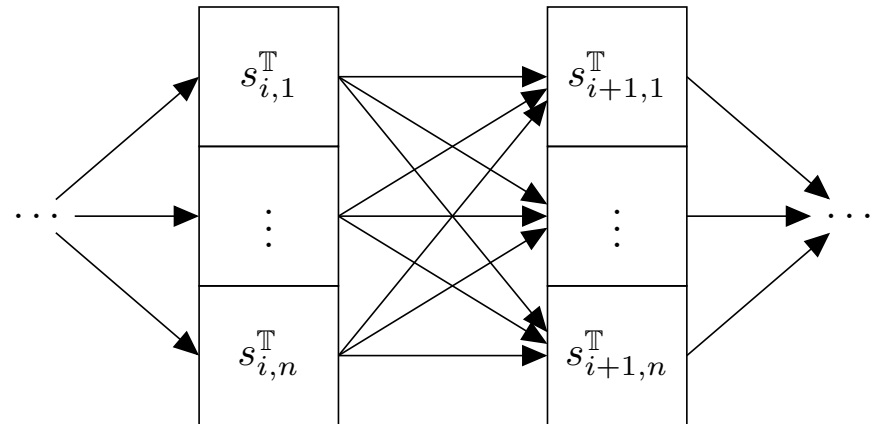


SMDP over Options

Plan:

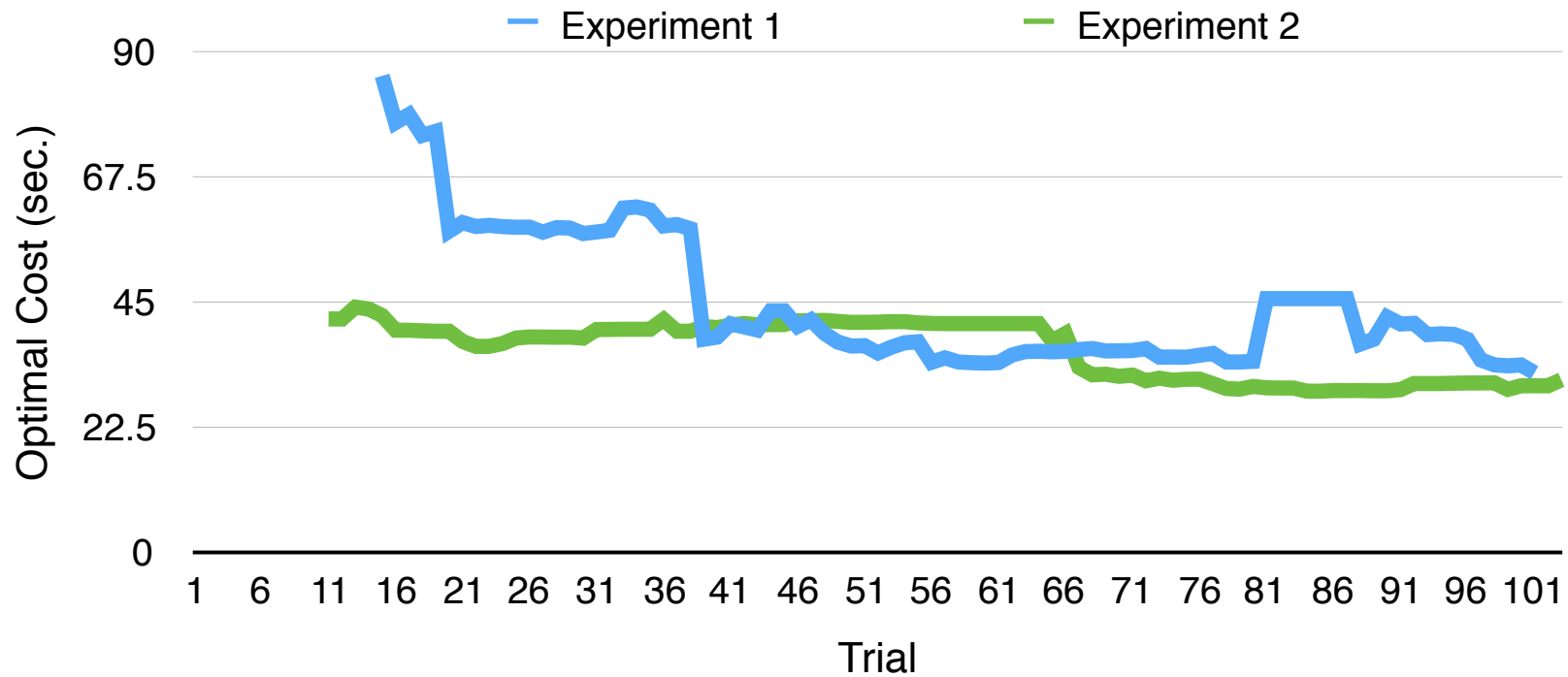


SMDP:



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# Planning into Reinforcement Learning



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# Examples of Learning



Reset  
loaded Plan/Controllers/MC with 9 actions  
Reset Map  
Reset





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# Examples of Learning

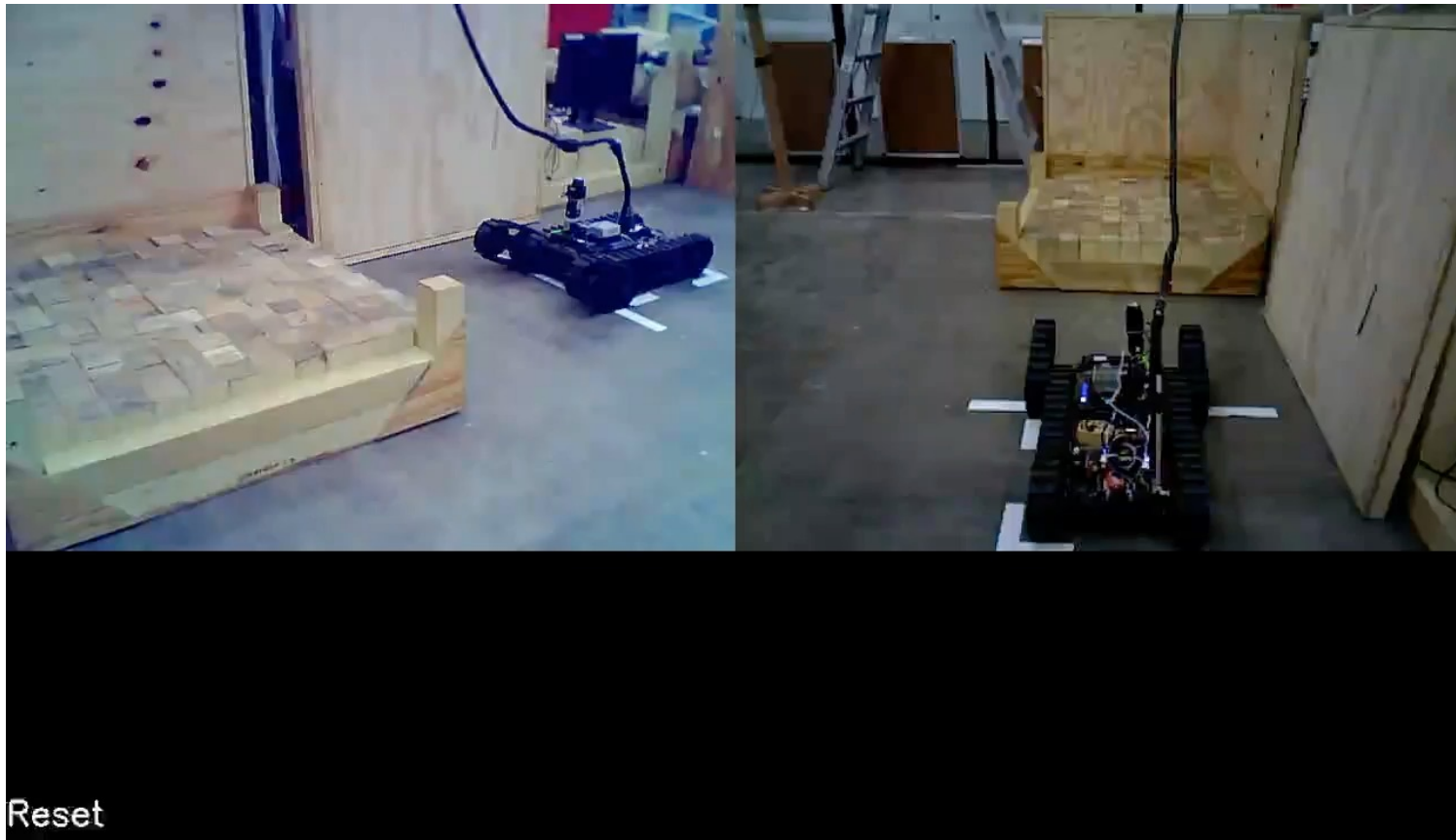


Reset  
loaded Plan/Controllers/MC with 10 actions  
Reset



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# Examples of Learning



**Noon Gudgin**

**Thank you**

**Day 3: Localisation,  
Mapping and Navigation**

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